Temperature Dependent Nucleon Mass And Entropy Bound

Inequality

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Abstract

Mass of a baryon as a function of temperature is calculated using colour-singlet partition function for massless quarks (with two flavours) and abelian gluons confined in a bag with a temperature dependent bag pressure constant $B(T)$. The non-perturbative aspect of QCD interaction is included through colour-singlet restriction on quark-gluon partition function in a phenomenological way. The entropy bound inequality $S/E \leq 2\pi R/\hbar c$, where S, E and R are entropy, energy and radius, respectively of the enclosed system with $\hbar c = 197.331$ MeVfm, is found to be consistent with the equilibrium solutions of the baryon mass upto a temperature T_E. There is a region of temperature $T_E < T < T_C$ (T_C is critical temperature for quark-gluon plasma formation) in which no admissible equilibrium states exist for the bag. We say that the system expriences a phase jump from hadron to quark-gluon plasma through thermodynamic non-equilibrium processes.

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Theory of black hole supplimented with the second law of thermodynamics could be judiciously applied to gain new insights into everyday physics - ordinary thermodynamics, particle physics and communication theory. The second law of themodynamics states that the entropy of a closed system tends to a maximum, but this law does not restrict the value of this maximum. Bekenstein [1] suggested on the basis of black hole thermodynamics [2] that if a physical system can be enclosed in a sphere of radius R, then there is an upper bound on the ratio of the maximum entropy S to its energy E given by

$$\frac{S}{E} \leq \frac{2\pi R}{\hbar c}$$

$$\text{or} \quad C = \frac{S}{E} - \frac{2\pi R}{\hbar c} \leq 0 \tag{1}$$

Many objections have been raised [3] regarding the original gedanken experiments involving thermodynamic systems which led to this conjecture of the bound (1). In early days there was no rigorous proof; rather over the years it has become a matter of general belief that the bound (1) is based on the variety of physical examples supporting it, particularly systems composed of free quantum fields [4] or many quantum mechanical particles [5]. In view of this bound (1) it was also possible to set limit on the number of generations of quarks [6]. Validity of this bound in systems with strong grivitional fields [7], with quartic field interaction [8] and superstring systems [9] gives us confidence on the correctness of this conjecture. Communication theory [6, 10] is also enriched with the bound putting limit on the rate at which information can be transferred in terms of the message energy in a communication channel. Furthermore, Bekenstein [11] has argued that the cosmological singularity in Friedmann model is inconsistent with bound (1), which could forbid the appearence of singularity from thermodynamic view point. A deductive proof of the bound (1) for free quantum fields enclosed in an arbitary cavity has been given by Schiffer [12, 13]. Very recently the bound (1) has been used to predict the limiting temperature for mesons formation in heavy ion collisions keeping the meson mass fixed at its bare $T = 0$ mass in spherical bag by Dey et al [14] and allowing the meson mass to depend on temperature in a self-consistent manner in a spherical as well as a deformed bag by Mustafa et al [15].

Now we would like to test the bound (1) for a nucleonic bag containing quarks and abelian gluons with temperature dependent bag pressure constant $B(T)$ and allowing the nucleon mass to depend on

temperature in a self-consistent manner. Before talking of calculational details let us first briefly present the outline of the mathematical steps. The colour-singlet grand canonical partition function [16-19] for a bag containing quark, antiquark and gluon is given by

$$\mathcal{Z}_0 \; = \; \int d\theta(g) \; e^{\Omega(\theta)} \tag{2}$$

where $d\theta(g)$ is the invariant Haar measure of colour $SU(3)$ group given as

$$\int d\theta(g) \; = \; \frac{1}{24\pi^2} \int_{-\pi}^{+\pi} d\theta_1 \; d\theta_2 \prod_{j<i}^{N_C} [2\sin\frac{1}{2}(\theta_j - \theta_i)]^2 \tag{3}$$

and

$$
\begin{aligned}
\Omega(\theta) \;\; = \;\; & \sum_i \sum_{K=1}^{\infty} \frac{1}{K} \{ [(N_C - 1) + 2\cos K(\theta_1 - \theta_2) \\
& + 2\cos K(2\theta_1 + \theta_2) + 2\cos K(\theta_1 + 2\theta_2)]e^{-K\beta\epsilon_i^g} \\
& + (-1)^{K+1}[\cos K\theta_1 + \cos K\theta_2 \\
& + \cos K(\theta_1 + \theta_2)][e^{-K\beta(\epsilon_i^q - \mu)} + e^{-K\beta(\epsilon_i^q + \mu)}]\},
\end{aligned}
\tag{4}
$$

where θ_i is invariant group parameter with $\sum_i^{N_C} \theta_i = 0$, N_C being the number of colour which is three here. ϵ_i^q and ϵ_i^g are, respectively, the quark and gluon single particle energies in the bag, μ is the chemical potential and $\beta = 1/T$, the inverse of temperature. Most of the physical quantities can be calculated using this $\mathcal{Z}_0$. Instead of going to continuum limit all quantities are computed by performing a finite summation over discrete states which ensures the finite size of the system [16,17,20]. The colour-singlet thermodynamic potential is given by

$$\Omega_0 \; = \; \ln(\mathcal{Z}_0) \tag{5}$$

One can get the net number of quarks (the excess number of quarks over antiquark) by adjusting chemical potential μ such that

$$N \; = \; T \frac{\partial \Omega_0}{\partial \mu} \; = \; N_q \; - \; N_{\bar{q}} \tag{6}$$

The total energy, E_T and free energy, F_T are, given by

$$E_T = T^2 \frac{\partial \Omega_0}{\partial T} + \mu N + B(T)V + \frac{d}{R} \qquad (7)$$

$$F_T = -T\Omega_0 + \mu N + B(T)V + \frac{d}{R} \qquad (8)$$

where V is the volume of the bag and d/R accounts for the centre of mass motion, gluomagnetic interaction and the Casimir energy without any explicit T dependence. In view of Aerts and Rafelski [21] we have fitted $d = -3.8274$ from the nucleon mass 938 MeV. The bag pressure $B(T)$ should decrease with increase of temperature. This was found from QCD sum rule by Dey et al [22], from NJL model by Li et al [23] and from Soliton bag model by Song et al [24]. We choose the simpler form given in ref.[22]

$$B(T) = B(0)\left[1 - \left(\frac{T}{T_C}\right)^4\right] \qquad (9)$$

We use the value of $T_C = 165$ MeV as obtained in ref.[16, 17]. At thermodynamical equlibrium the pressure $P = -\left(\frac{\partial F_T}{\partial V}\right)_{T,N}$ is balanced by the bag pressure which leads to the equilibrium energy E of the bag as

$$E = T^2 \frac{\partial \Omega_0}{\partial T} + \mu N + \frac{d}{R} = 3B(T)V \qquad (10)$$

and then eq. (7) reduces to

$$E_T = M(T) = 4B(T)V \qquad (11)$$

The entropy of the system is given as

$$S = -\left(\frac{\partial F_T}{\partial T}\right)_V \qquad (12)$$

Minimizing the free energy with respect to radius at a given value of temperature stable solutions of the bag can be obtained. This can be done graphically plotting F_T vs R as in ref. [16, 17]. Such a plot is displayed in fig.1 with $B^{1/4}(0) = 200$ MeV for $T = 142$, 143, 144, and 144.3 MeV. We must mention that at each point the chemical potential μ is adjusted such that the constraint (6) with $N = 3$ is satisfied. As seen in fig.1 the first three curves have two extremum points; a minimum at a

smaller value of R and a maximum at a larger R. Physically a meaningful maximum occurs at $R = 2.5$ fm with $\mu \sim 0$ for T (say T_S) $= 130$ MeV (not shown in the fig. 1). The significance of this value is that for $T = T_S$ a metastable solution appears corresponding to the maxima [17]. If the value of $T < T_S$ MeV, there is essentially one extremum (minimum) in F_T, and we have a stable nucleon. Finally at $T_E = 144.3$ MeV $> T_S$, the maximum almost disappears and both the extrema meet in one point. Even for a slightly higher value of T than T_E, there is no equilibrium solutions of the bag implying thermodynamical instability of the system. This persists in the temperature $T_E < T < T_C$ because at T_C, $B(T) = 0$ implies that there is no bag boundary as $R \to \infty$.

As given by relation (11) the temperature dependent mass $M(T) = 4B(T)V$ at the extremum (with respect to R) points of F_T. Equivalently one can also try to match the right hand sides of eq. (7) and eq.(11) as a function of R keeping in mind that the quark number constraint (6) has to be always satisfied. In this way we can calculate $M(T)$ and a corresponding $R(T)$. At such points the bound inequality (1) is always satisfied. In principle corresponding to each extremum point of F_T the inequality (1) is satisfied upto $T = T_E$ and finally at $T > T_E = 144.3$ MeV the right hand sides of eq.(7) and eq.(11) can not be matched and, therefore, the value of the C in eq.(1) can not be computed. Then we may say that the system has reached a themodynamic non-equilibrium phase. Thus we find that the inequality (1) is fully consistent with the equilibrium solutions of F_T in a variational sense and this T_E is called the maximum temperature at which baryon can exist in the thermodynamic equilibrium state. In fig.2 we display a plot of $C(T)$ vs T where $C(T)$ is evaluted at the extremum points of F_T as given in fig.1. We notice that $C(T)$ is always negative and, obviously, has two values in the temperature region $T_S < T < T_E$. $|C(T)|$ larger negative corresponding to the maximum. This curves reminds us of the $\mu - T$ plot in ref. [17] and the branch of C for $T_S < T < T_E$ corresponds to a metastable region.

The temperature region $T_E < T < T_C$, in which there is no thermodynamic equilibrium states, corresponds to a non-equilibrium phase jump of the system. Within the present model two scenarios are then possible. Suppose that the system is in the state of thermodynamic equilibrium and the nucleons temperature is raised to T_S such that it can absorb enough latent heat and move to the metastable state. Then depending upon the dynamical conditions temperature reaches T_E and the

system may go to quark-gluon phase at T_C through non-equilibrium processes. Or conversely we may say that heavy ion collisions do not have enough time to thermalize the hadrons and they can jump to quark-gluon plasma phase through non-equilibrium processes.

In fig.3 we have shown a plot of $M(T) = 4B(T)V$ vs T. We find that $M(T)$ increases with the increase of T with the rate of increase being very slow for $T < 120$ MeV. For $T \geq T_S$, $M(T)$ increases very rapidly. At $T = T_S = 130$ MeV there are two values; one with smaller value of $M(T) = 1107$ MeV and other one with larger value of $M(T) = 33349$ MeV (not shown in fig.3), the latter corresponding to a metastable state. At $T_E = 144.3$ MeV two values coincide to one value of $M(T) = 1919$ MeV. In finite temperature field theory [25] there are two types of masses; one which appears as the pole of the propagator (the Green's function) of the field, and the other appears in the Yukawa type fall of the correlators of the currents. The former is sometimes called the pole mass and the latter is often known as the screening mass. Of course, both go over to the unique physical mass in the zero temperature limit. The masses measured by Gottlieb et al [26] are the screening masses at finite temperature. It is found that if there is no interaction among the quarks then the self-energy of the quarks would be screened by the Matsubara frequency πT. Then the meson screen masses would go as $2\pi T$ and the baryon as $3\pi T$ around the critical temperature. We find that in a simple bag model like picture the baryon mass goes like $4\pi T$ around T_E (144 MeV). Thus, in a simple minded bag model description the dependence of the baryon mass on temperature is rather satisfactory. Although there is no obvious connections, the obtained $M(T)$ in the bag model is intriguingly close to that of lattice calculations.

We may point out another outcome of the present calculations. As seen above, using a temperature dependent bag pressure constant $B(T)$ with a colour-singlet partition function, we find an equilibrium temperature $T_E \approx 145$ MeV when a temperature independent $B^{1/4}(0) = 200$ MeV is used. The value of $T_S = 130$ MeV for colour-singlet case. If, on the other hand, a colour **unprojected** calculation is performed, with $B(T)$ using $T_C = 145$ MeV [17], the new value of $T_S = 121$ MeV and $T_E = 125$ MeV. Thus, $T_E - T_S = 4$ MeV for colour unprojected case and about 15 MeV for the colour-singlet case. This marked difference in the temperature width of metastable states is due to the non-perturbative

aspect of the QCD interaction and is almost similar to as that found in ref. [17] when temperature independent $B^{1/4} = 200$ MeV was used.

Finally we would like to draw the following conclusions: Using a temperature independent $B^{1/4} = 200$ MeV earlier (ref. [17]) we had found $T_C = 165$ MeV. Then using a temperature dependent $B = B(0)[1 - (T/T_C)^4]$ with $B^{1/4}(0) = 200$ MeV and $T_C = 165$ MeV a equilibrium temperature $T_E \approx 145$ MeV is found. The temperature window of metastable states $T_E - T_S$ is even now (with $B(T)$) about 15 MeV for colour-singlet case and about 4 MeV for the colour unprojected case.

For the temperature range T_S to T_E the baryon mass increases rapidly with the increase of T, the dependence being roughly as $4\pi T$. This is qualitatively consistent with the so called screening mass [26].

The entropy bound inequality $C < 0$ (eq.1) is satisfied at all extremum points of F_T as a function of R. C does not become positive but it is close to zero ($|C|$ smallest) at $T \sim 140$ MeV. For $T > T_E$ the eqs. (7) and (11) can not be solved self-consistently except at $R \to \infty$ and, therefore, C can not be computed. This implies that hadrons can exist in thermodynamic equilibrium states only upto a temperature T_E. Thus, this inequality can certainly be used as a criterion for testing a system to be bound.

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FIGURE CAPTIONS

Figure 1: Variation of the colour projected free energy, F_T as a function of bag radius, R.

Figure 2: Variation of entropy bound inequality, C at the extremum points of F_T, with temperature, T.

Figure 3: Variation of self-consistently obtained temperature dependent nucleon mass, $M(T)$ as a function of temperature, T.